ANALYSIS OF DISPERSION IN ARBITRARILY CONFIGURED DIELECTRIC-LOADED TRANSMISSION STRUCTURES

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Abstract

An accurate solution to the problem of wave propagation along arbitrarily-configured transmission media composed of conductors and/or inhomogeneous dielectrics is presented. The solution is available in the form of a digital computer program, which yields computed cutoff frequencies and dispersion characteristics for both fundamental and higher-order modes. Sample calculations are presented for shielded microstrip, to illustrate the validity of the analysis, and channelized suspended microstrip. The solution approach presented is distinctive in its ability to easily treat complicated, salient features of arbitrary cross sections, as is typified by the channel in the suspended-microstrip example.

I. Introduction

The success of microstrip in microwave integrated circuit applications has prompted increased interest in employing integrated circuit technology (thin film and photolithography) for applications at higher microwave and millimeter-wave frequencies. Because microstrip becomes discouragingly lossy and difficult to fabricate at these higher frequencies, attention has focused on configuring and exploiting new transmission media for these applications. At these higher frequencies problems related to dispersion and higher-order modes in inhomogeneous planar transmission media become much more significant than at lower microwave frequencies. The computer-aided analysis described in this paper can provide this much-needed design information for planar transmission media which employ composite conductor and/or dielectric materials. Dispersion and higher-order mode characteristics can be determined for a wide variety of transmission structures having different geometries and material parameters, including practical effects due to packaging enclosures.

II. Formulation

The problem of electromagnetic wave propagation along uniform, guiding structures composed of mixed dielectrics and conductors within a conducting enclosure is considered. The solution is suitable for treating enclosures containing a finite number, N_D, of discrete homogeneous, isotropic dielectric regions as is shown in the generic cross section shown in Fig. 1. The solution is effected by invoking the equivalence principle,[1] wherein each homogeneous dielectric region is bordered by a set of electric ($\vec{J}$) and magnetic ($\vec{M}$) current source distributions as is depicted symbolically in Fig. 2 for only the kth dielectric region from Fig. 1. In Fig. 2 the dielectric medium for this region (characterized by permittivity ε_k) is fictitiously extended to all space. Combinations of magnetic and electric current sources (to be determined) are conceptually placed along the contour C_k which surrounds the kth region and are required to give to electromagnetic fields which are zero everywhere outside C_k and identical to the fields $\vec{E}_k$ and $\vec{H}_k$ at each point within the interior of region k for the original problem shown in Fig. 1. The electric and magnetic fields at any point in the cross section shown in Fig. 2 can be written as

$$-\vec{E}_k = \frac{1}{\varepsilon_k}\nabla \times \vec{F}_k + j\omega\vec{A}_k + \nabla(\phi_e)_k \qquad (1)$$

$$-\vec{H}_k = -\frac{1}{\mu_o}\nabla \times \vec{A}_k + j\omega\vec{F}_k + \nabla(\phi_m)_k \qquad (2)$$

where $k = 1, 2, \ldots, N_D$. Here, $\vec{A}_k$ and $(\phi_e)_k$ are the vector and scalar potentials, respectively, due to the electric current sources residing along C_k. Similarly, $\vec{F}_k$ and ϕ_m are the vector and scalar potentials, respectively, due to the magnetic current sources residing along C_k. μ_o is the free-space permeability.

The following boundary conditions are imposed for each of the N_D dielectric regions in the problem at hand:

$$(\vec{E}_{ki})_{tan} = 0 \qquad \text{conductor surface} \qquad (3)$$

$$(\vec{E}_{ki})_{tan} - (\vec{E}_{ka})_{tan} = 0 \qquad (4)$$

$$(\vec{H}_{ki})_{tan} - (\vec{H}_{ka})_{tan} = 0 \qquad \text{dielectric-dielectric interface} \qquad (5)$$

$$(\vec{E}_{ko})_{tan} = 0 \qquad k = 1,2, \ldots, N_d \qquad (6)$$

$(\vec{E}_{ki})_{tan}$ is the component of electric field tangential to and just inside C_k at any point along conducting surface. $(\vec{H}_{ki})_{tan}$ is the component of magnetic field which is tangential to and just inside C_k at any point which had a dielectric-dielectric interface. $(\vec{E}_{ka})_{tan}$ and $(\vec{H}_{ka})_{tan}$ are components of electric and magnetic fields, respectively, which are tangential to a dielectric-dielectric interface lying along C_k but at points just inside a different dielectric medium adjacent to the kth medium. Finally, $(\vec{E}_{ko})_{tan}$ is the component of electric field tangential to and just outside a dielectric-dielectric interface lying along C_k, but with the dielectrics in this "outside" region replaced by the same material existing on the interior of region k. Eqn (6) is not imposed at points near conducting surfaces lying along C_k because only electric current sources are postulated along these surfaces for each of the N_D regions; hence, eqn (6) is assured due to eqn (3).

III. Reduction to Matrix Formulation

The problem is reduced to matrix form using a moment solution.[2] The tangential field components appearing in eqns (3) - (6) can each be expressed as linear, integro-differential operators operating on vector sources for each k ($k = 1, 2, \ldots, N_D$). It is to be noted that the sources for tangential components of $\vec{E}_{ki}$, $\vec{H}_{ki}$, and $\vec{E}_{ko}$ are only those lying along C_k, whereas sources for tangential components $\vec{E}_{ka}$ and $\vec{H}_{ka}$ lie in part along C_k and in part not along C_k. The reduction to a matrix formulation is achieved by employing a method of moments solution using: a pulse-function[3] expansion of the equivalent sources, point-matching[4] of the boundary conditions, and an approximation of the operator.[5] Applying this procedure for each of the N_D regions, a system of equations is obtained and then reduced to the following matrix equation by eliminating dependent variables and associated equations.

$$[H]\,[S] = [0] \qquad (7)$$

Here, the elements of the matrix [H] are determined purely by the guiding structure geometry, material parameters, and the moment solution expansion and testing functions. The details of this procedure and the explicit form of the elements of [H] will be given in a future publication.

IV. Determination of Cutoff Frequencies and Phase Constants

If a complete expansion function set on the domain of the operator is used in the development of equation (7) then solutions exist if and only if

$$|\text{Det } H(f, k_z)| = 0 \qquad (8)$$

Since Det H is, in general, a complex quantity, then equation (8) infers

$$\text{Re}[\text{Det } H\ (f, K_z)] = 0 \qquad (9)$$

$$\text{Im}[\text{Det } H\ (f, k_z)] = 0 \qquad (10)$$

In equations (8) - (10), f is the operating frequency and k_z is the phase constant in the direction of propagation along the guiding structure. In general, the expansion set used here will enable only an approximation to the exact source currents. For an adequate approximation solutions are characterized by

$$\text{Det}|H(f, k_z)| = \text{Minimum} \qquad (11)$$

Also due to the approximations used, for a fixed frequency f, the values of k_z needed to satisfy equations (9) and (10) are, in general, slightly different. Values of k_z are determined for any f by determining solutions to equations (1)-(11). Since the guiding structure is enclosed within a conducting package, cutoff frequencies for all possible modes correspond to solutions of equations (9)-(11) for $k_z = 0$.

V. Examples

A. Microstrip in a Conducting Box

To illustrate the validity of the solution method described here, results computed for the shielded-microstrip cross section shown in Fig. 3 are plotted in Fig. 4. It is observed that the calculated fundamental mode dispersion characteristic agrees reasonably well with results due to Mittra and Itoh.[6] The cutoff frequencies and phase constants for the even-symmetry higher order modes agree well with results due to Yamashita and Atsuki.[7] For reference, phase constants are plotted for the cases of TEM propagation in air and dielectric material (ε_r=8.875), respectively. Although only even-symmetry higher order modes are shown in Fig. 4, the solution method described here determines dispersion characteristics for all modes, symmetric or otherwise.

B. Channelized, Suspended Microstrip

The cross section of the structure treated in this section is shown in Fig. 5. The channel located above the conducting strip in this figure is used for the suppression of higher-order mode propagation. Additional potentially useful features of this transmission structure are: (1) wider strip widths for 50 Ω impedance levels with associated mitigation of fabrication problems and (2) reduced dissipation losses.[8] The evaluated dispersion characteristic of the fundamental mode for this structure is shown in Fig. 6. Plotted for reference are the associated TEM phase constants in air and dielectric material (ε_r = 10.0), respectively. Cutoff frequencies for the first and second higher-order modes are found to be 17.2 GHz and 27.4 GHz, respectively. It should be noted that the formulation employed in this work allows for great flexibility in the variety of structures treated. This permitted the proper characterization of the ground plane channel encountered in this example.

VI. Discussion

The analysis method presented here fulfills several criteria[9] established for assessing the utility of such methods for solving the problem of propagation along arbitrarily-configured wave-guiding structures. Consistent with this assessment scheme, several features can be attributed to the method presented here. This method is:

1. Effective for wave-guide structures having different shapes and materials.
2. Adaptable to implementation by computer program.
3. Capable of computing dispersion characteristics and cutoff frequencies for the first several modes in addition to the dominant mode.
4. Sufficiently accurate for most design applications.
5. Available in the form of a working computer program.

Several other attributes are worth noting. This computer-aided analysis has been implemented to account for effects on propagation due to an arbitrarily shaped conducting enclosure. Coupled transmission lines within the same enclosure can also be analyzed. Finally, although the formulation presented in this paper does not explicitly indicate, the analysis can be easily extended to determine other propagation characteristics, including: electric and magnetic field distributions, modal currents, impedance parameters, and dissipation losses.

References

1. R. F. Harrington, Time-harmonic electromagnetic fields, McGraw-Hill Book Co., pp. 106-10, 1961.
2. R. F. Harrington, Field computation by moment methods, Macmillan Co., pp. 126-31, 1968.

3. Ibid., Chapter 1, p. 11.

4. Ibid., Chapter 1, pp. 9-11.

5. Ibid., Chapter 1, pp. 14-5.

6. R. Mittra and T. Itoh, "A new technique for the analysis of the dispersion characteristics of microstrip lines," IEEE Trans. Micr. Th. Techs., Vol. MTT-19, No. 1, January 1971, pp. 47-55.

7. E. Yamashita and K. Atsuki, "Analysis of microstrip-like transmission lines by nonuniform discretization of integral equations," IEEE Trans. Micr. Th. Techs., Vol. MTT-24, April 1976, pp. 195-200.

8. B. E. Spielman, "Computer-aided analysis of dissipation losses in isolated and coupled transmission lines for microwave and millimeter-wave integrated-circuit application," NRL Report #8009, Naval Research Laboratory, Washington, D. C., July 1976, p. 28.

9. B. E. Spielman, "Waveguides of arbitrary cross section by solution of a non-linear integral eigenvalue equation," IEEE Trans. Micr. Th. Techs., Vol. MTT-20, No. 9, September 1972, pp. 584-5.

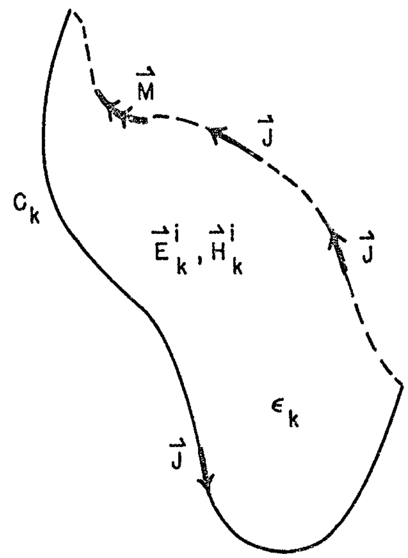

Fig.2 Conceptual Treatment Of Currents And Fields For kth Dielectric Region

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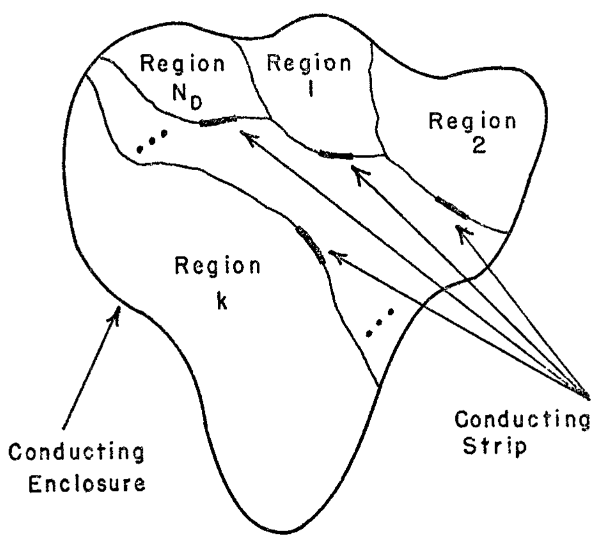

Fig.1 Generic Cross Section Of Mixed Conductors And Dielectrics in Conducting Enclosure

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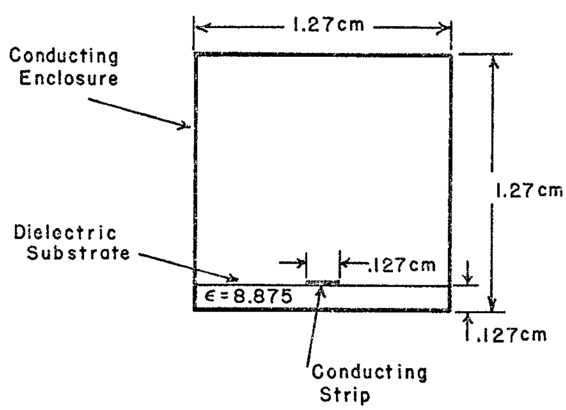

Fig.3 Cross Section For Microstrip In A Conducting Box

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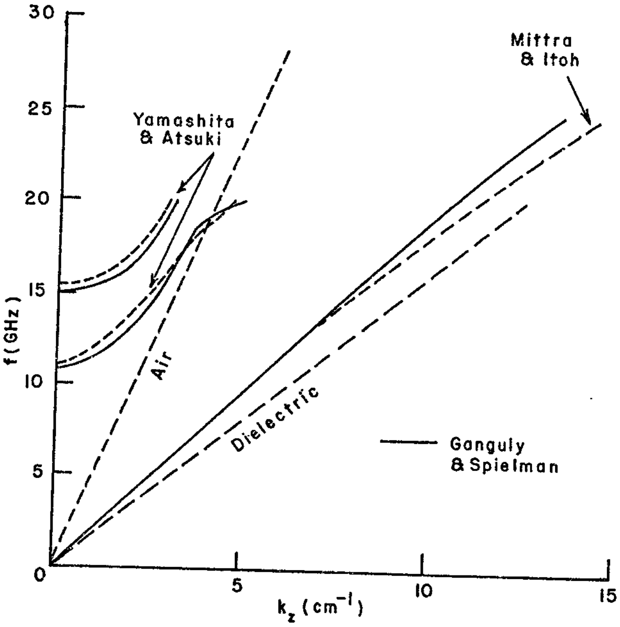

Fig. 4 Dispersion Characteristics For Microstrip In A Conducting Box

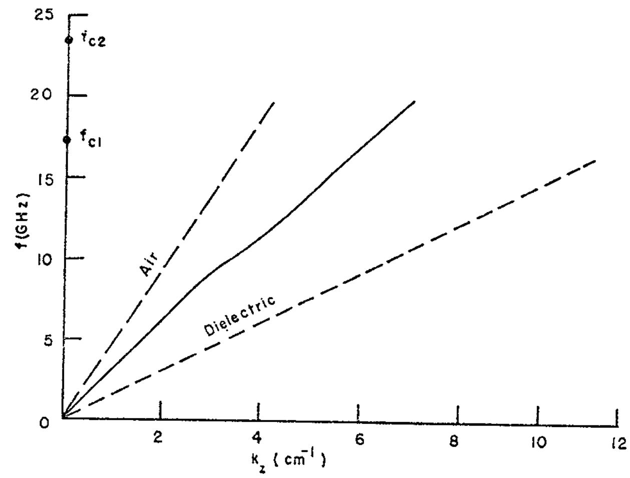

Fig. 6 Dispersion Characteristics For Channelized, Suspended Microstrip

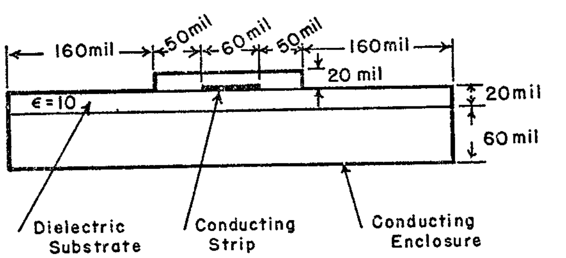

Fig. 5 Cross Section For Channelized, Suspended Microstrip